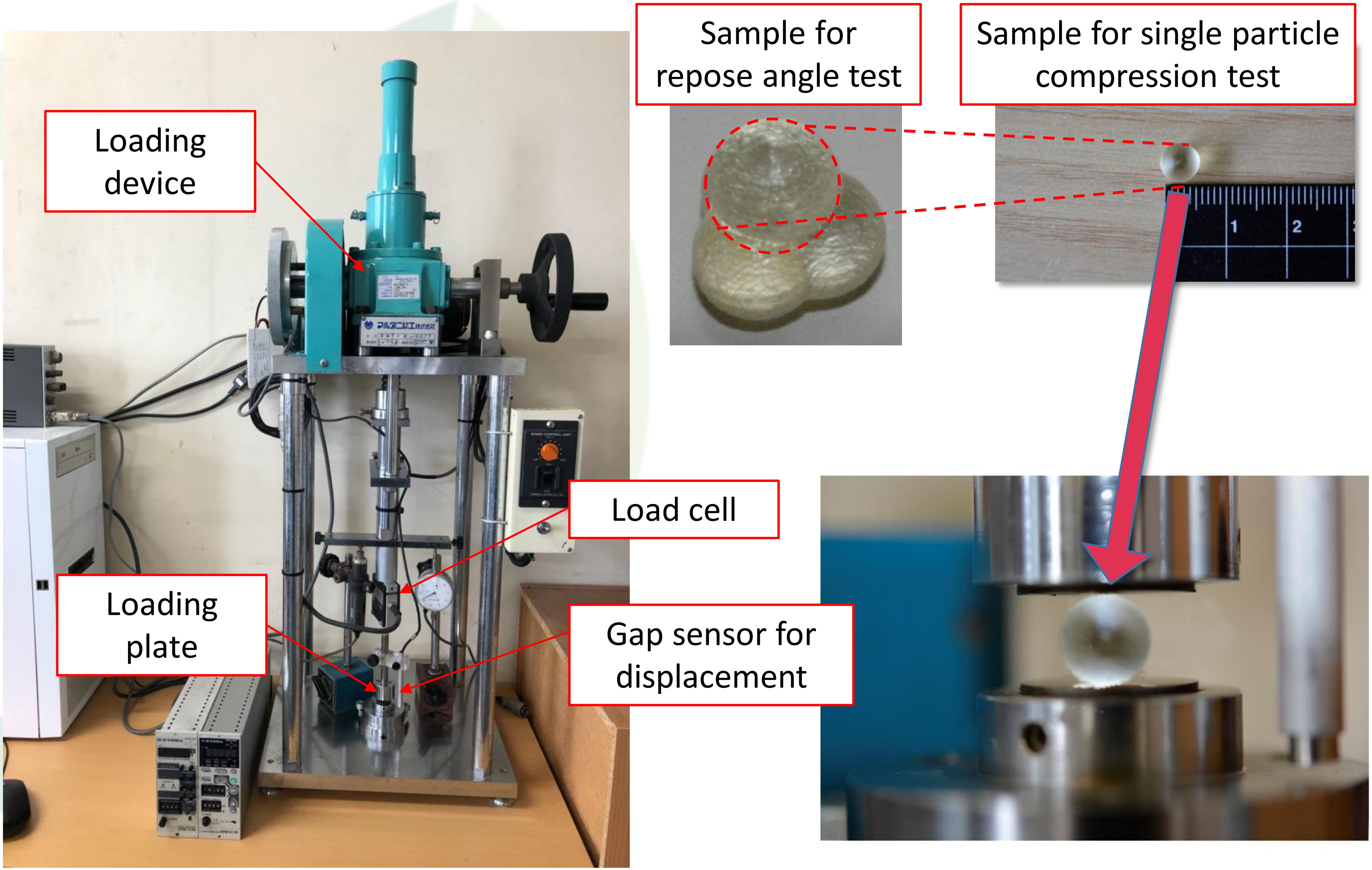
Single particle compression test

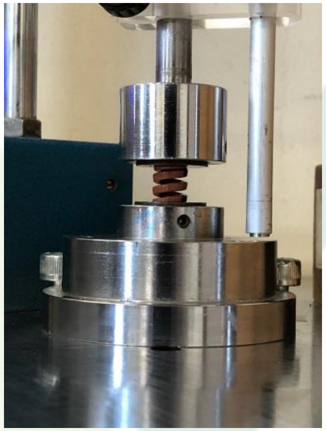
- The material describes the test procedure and summarizes the results.
- The test can be interpreted to determine the elastic shear modulus G (MPa). The average G value was 560 MPa (Standard deviation =158 MPa)
- For a normal Force of 0.1N, the contact stiffness in the normal direction, Kn was calculated as 6.02 $\times 10^4$ N/m (Standard deviation = 1.120 $\times 10^4$ N/m)
- If two springs are applied to a contact point, each spring coefficient is 1.20 $\times 10^5$ N/m.

- O'Sullivan, C., 2011, Particulate Discrete Element Modelling, pp.103-106.

Equipment for the single particle compression test



Transducers and Procedures



Load cell:

maximum = 100N

resolution =0.1N

Gap sensor for displacement:

max. = 1mm

resolution =0.4μm

Material:

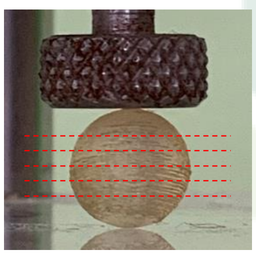
20 Spherical particles

(Each tetrahedral particle consists of 4 of these spherical particles)

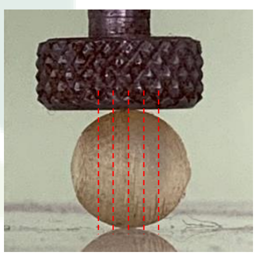
Direction of particle laminated surface:

Case where the laminated surface is horizontal (h-case)

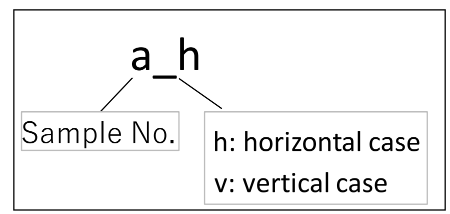
Case where the laminated surface is vertical (v-case)



Horizontal case



Vertical case



Compression speed:

about 30mm/min

Loading applied:

subjected to three cycles of loading and unloading process

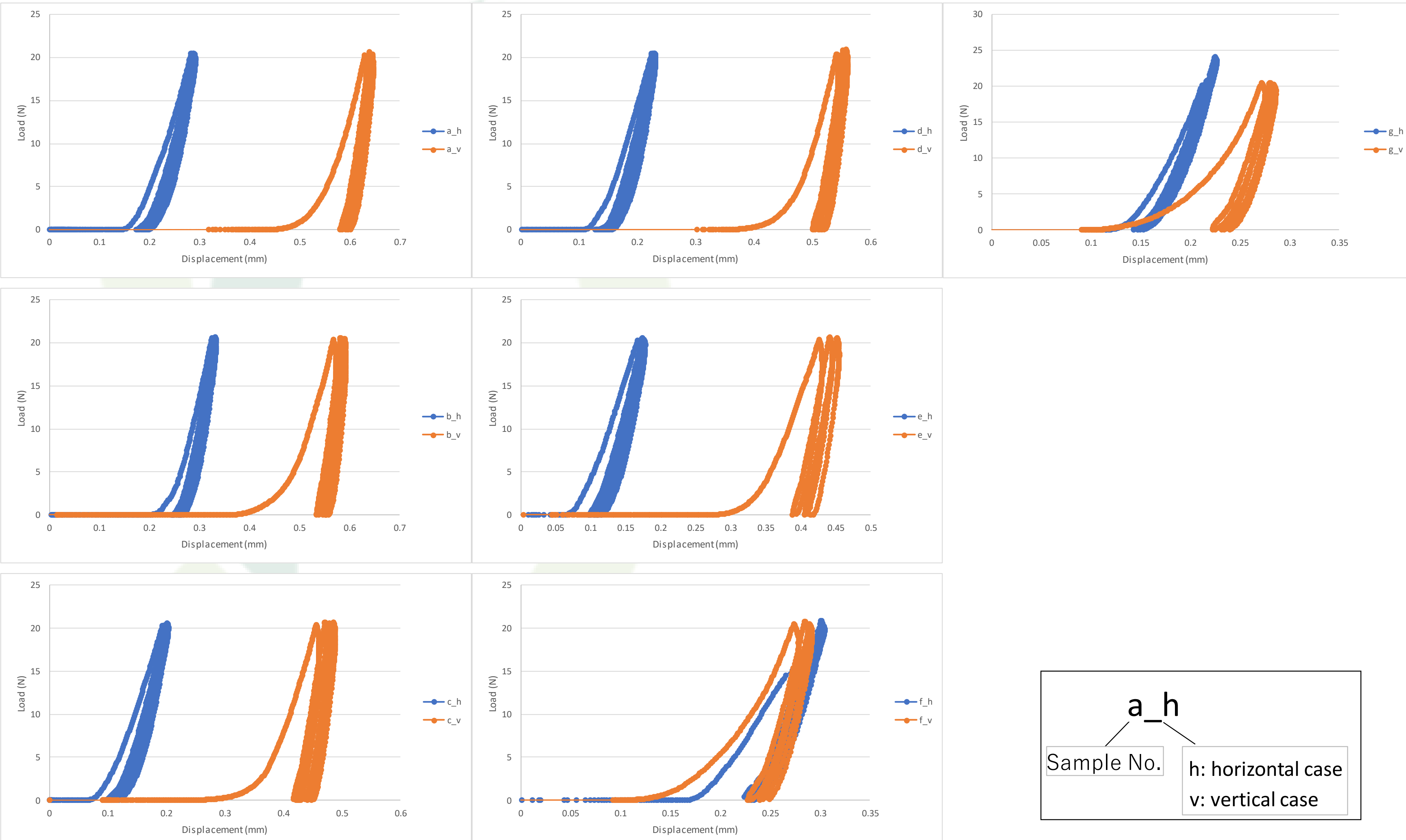
List of 20 spherical particles and its height

Sample No.	Before				After				Remarks
	Height_h	Height_v	Height_r	Height_average	Height_h	Height_v	Height_r	Height_average	
a	6.40	6.40	6.40	6.40	6.58	6.41		6.50	The size of particle before loading is average size of excluding a to e and s samples.
b	6.40	6.40	6.40	6.40	6.58	6.45		6.52	
c	6.40	6.40	6.40	6.40	6.56	6.41		6.49	
d	6.40	6.40	6.40	6.40	6.56	6.41		6.49	
e	6.40	6.40	6.40	6.40	6.59	6.41		6.50	
f	6.42	6.40	6.37	6.40	6.38	6.41	6.41	6.40	
g	6.40	6.42	6.42	6.41	6.46	6.44	6.35	6.41	
h	6.40	6.41	6.42	6.41	6.55	6.38	6.37	6.43	
i	6.38	6.37	6.36	6.37	6.31	6.34	6.33	6.33	The bottom is unstable and slips
j	6.54	6.40	6.34	6.43	6.35	6.39	6.37	6.37	
k	6.39	6.44	6.41	6.41	6.42	6.41	6.40	6.41	
l	6.42	6.38	6.36	6.39	6.35	6.34	6.35	6.35	
m	6.52	6.40	6.39	6.44	6.45	6.43	6.45	6.44	The bottom is unstable and slips
n	6.49	6.41	6.36	6.42	6.52	6.38	6.40	6.43	
o	6.35	6.35	6.39	6.36	6.40	6.31	6.36	6.36	
p	6.36	6.35	6.41	6.37	6.45	6.34	6.36	6.38	
q	6.54	6.44	6.38	6.45	6.38	6.41	6.36	6.38	
r	6.39	6.30	6.29	6.33	6.43	6.35	6.43	6.40	
s	6.40	6.40	6.40	6.40	6.45	6.37	6.40	6.41	The size of particle before loading is average size of excluding a to e and s samples.
t	6.42	6.39	6.42	6.41	6.42	6.41	6.38	6.40	The bottom is unstable and slips
Height average (Excluding a to e, s)	6.43	6.39	6.38	6.40	6.42	6.38	6.38	6.39	
Height average	6.42	6.39	6.39	6.40	6.46	6.39	6.38	6.42	

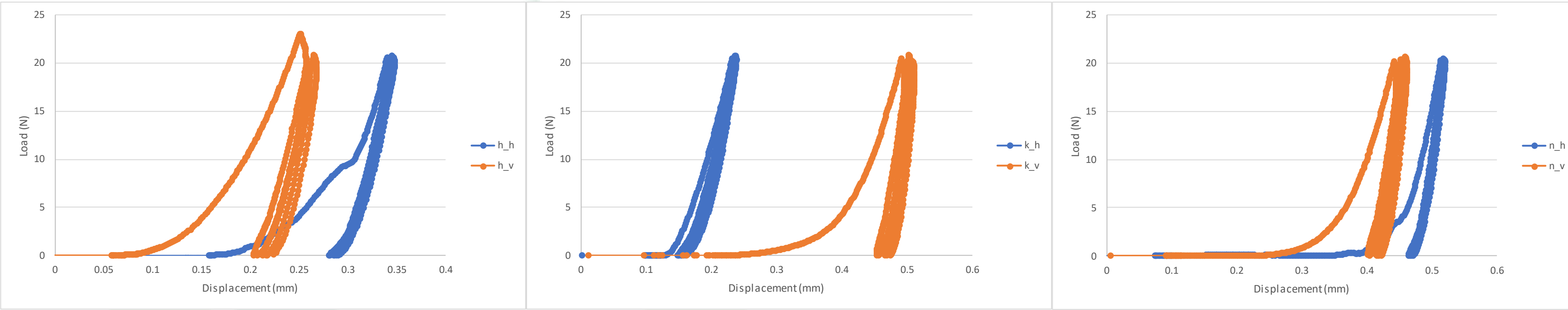
*unit : (mm)

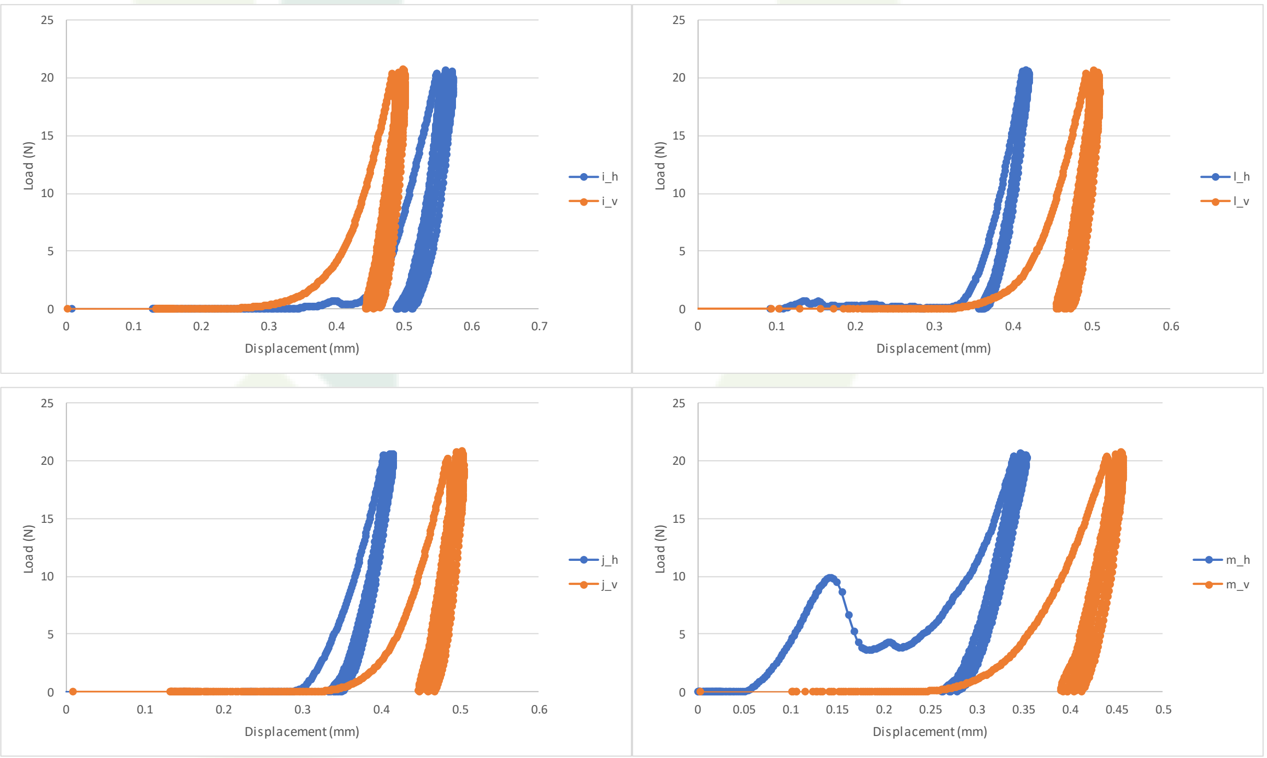
Height_h	Laminated surface is horizontal
Height_v	Laminated surface is vertical
Height_r	Random except above

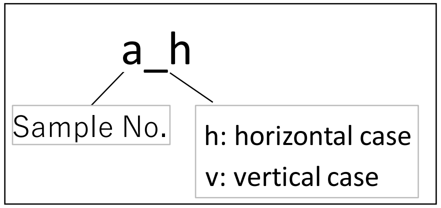
Load displacement relation No.1



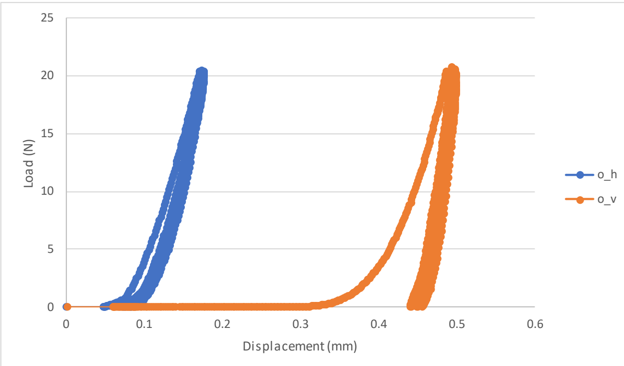
Load displacement relation No.2

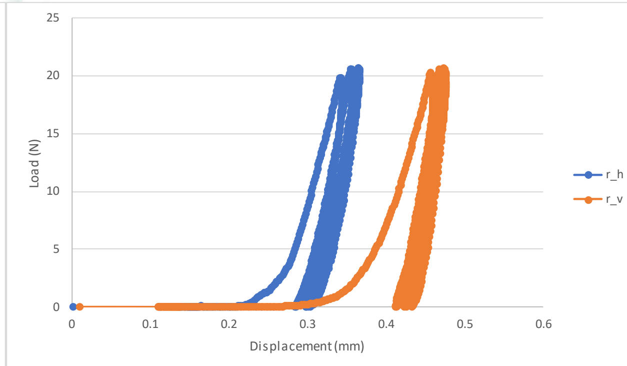


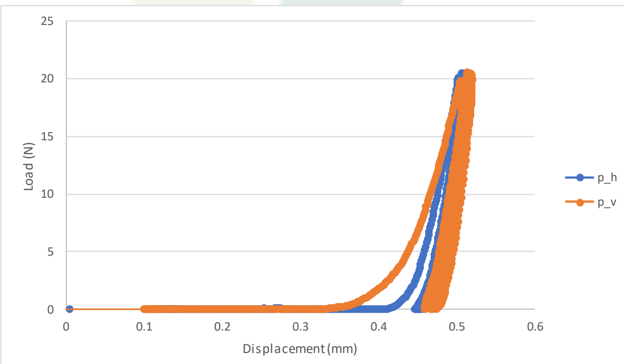


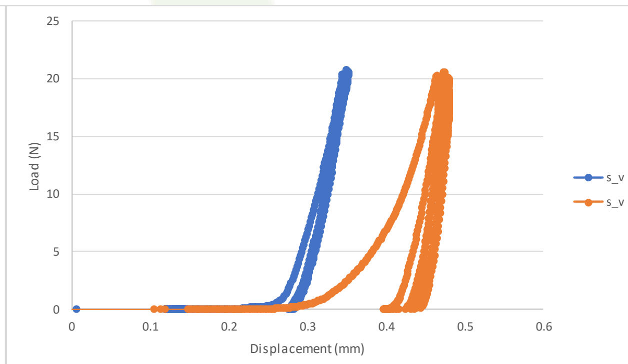


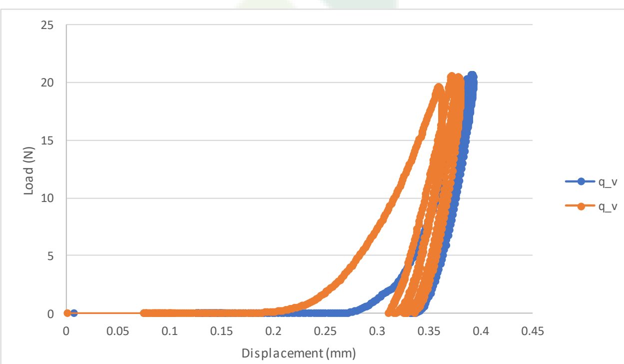
Load displacement relation No.3

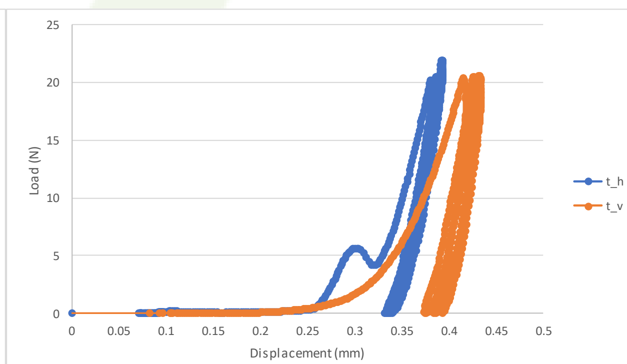


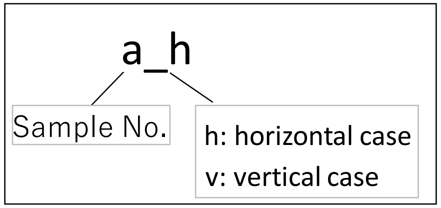




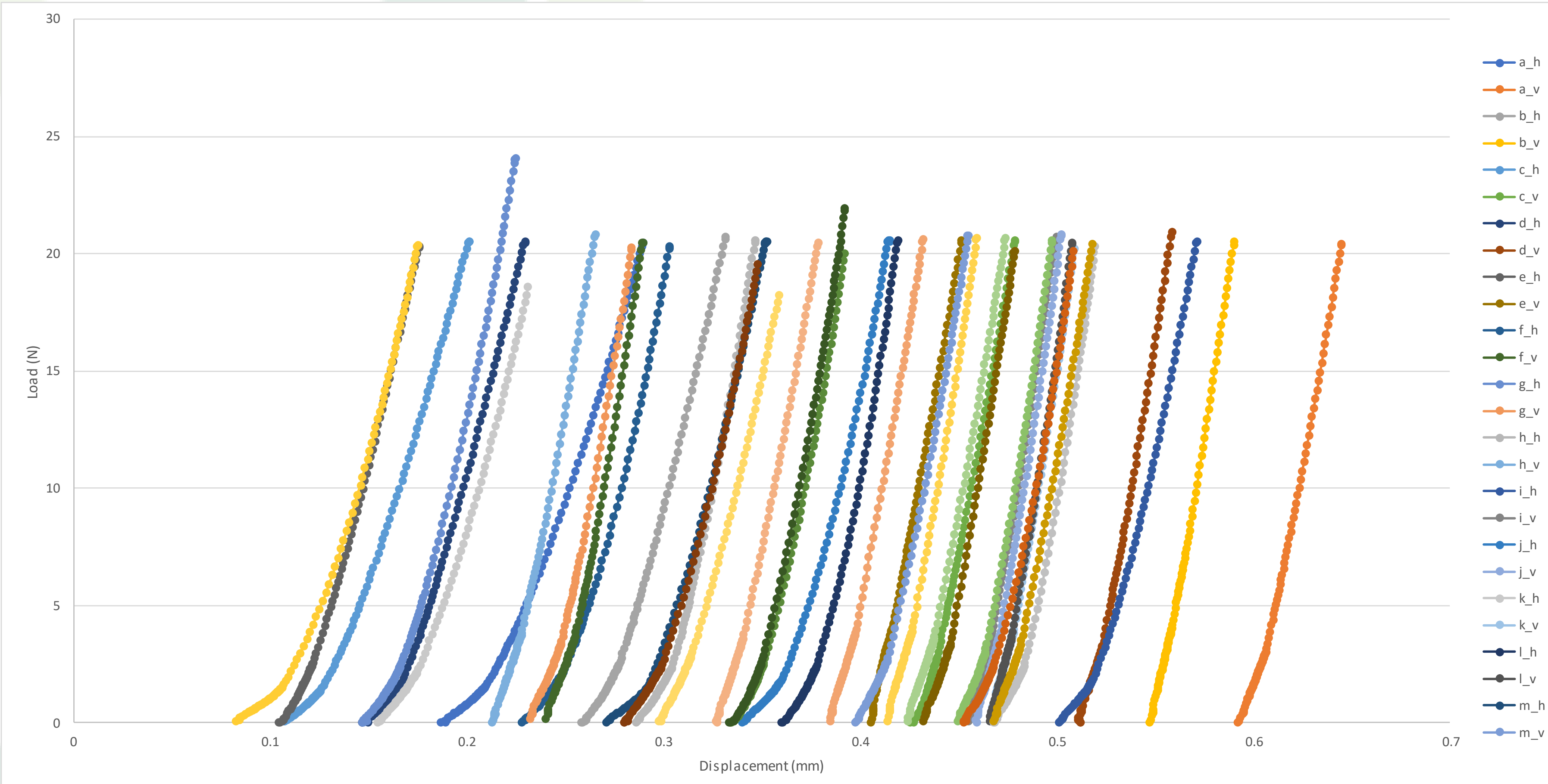








Load displacement relation for 3rd load cycle



Determining the elastic shear modulus using simplified Hertzian contact theory

The normal contact stiffness is given by $K_n = \left(\frac{2\langle G\rangle\sqrt{2\tilde{R}}}{3(1-\langle\nu\rangle)}\right)\sqrt{\delta_n}$ (1)

The normal contact force is calculated as $F_n = K_n\delta_n$ (2)

where δ_n is the sphere overlap and is equivalent to the displacement obtained the test.

For a sphere-boundary contact, the coefficients are given by R̃=R_sphere, ⟨G⟩=G_sphere, ⟨υ⟩=ν_sphere, where G is the elastic shear modulus, ν is Poisson's ratio, R is the sphere radius.

Eq.(2) for the normal contact force can be applied to the load-displacement relation for the 3 rd load cycle process. The value of G is obtained approximately. Then the G obtained can used to calculate to by the following equation using Poisson's ratio.

$$E = 2G(1+\nu)$$

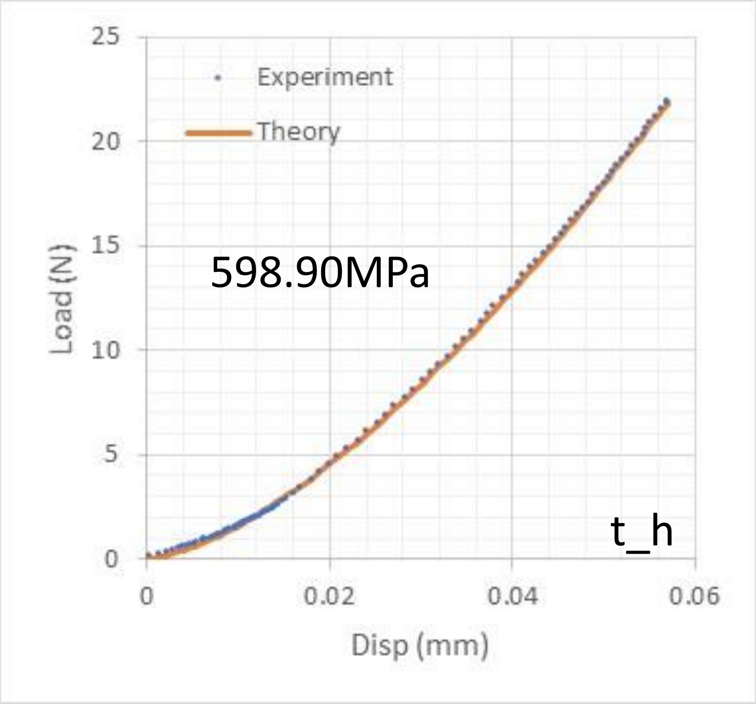
where ν is 0.37 as nominal value.

List of elastic shear moduli obtained from theory

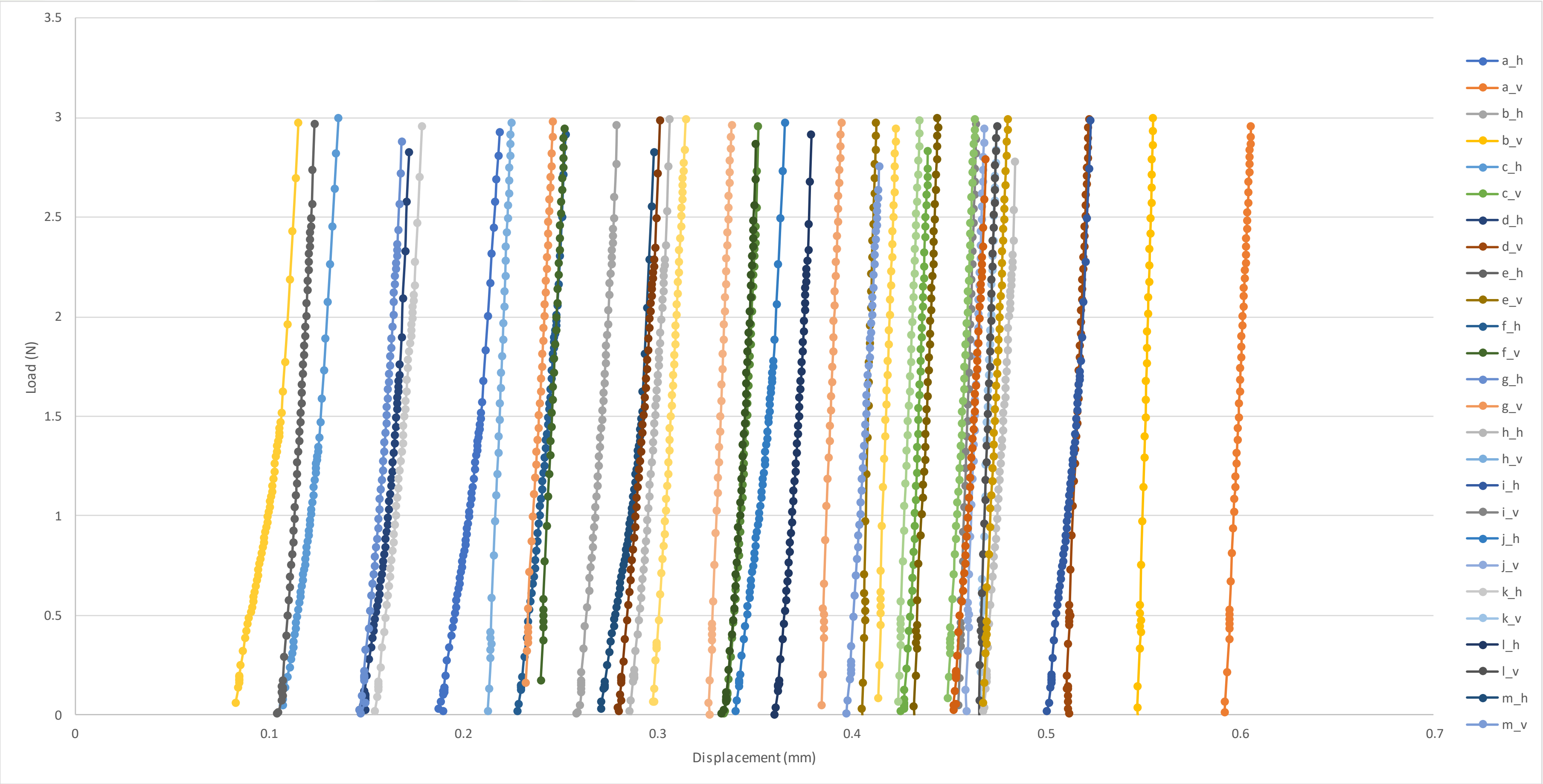
Sample name	Shear Modulus
a_h	250.96 MPa
b_h	402.94 MPa
c_h	280.95 MPa
d_h	361.04 MPa
e_h	401.94 MPa
f_h	392.55 MPa
g_h	435.40 MPa
h_h	581.54 MPa
i_h	452.11 MPa
j_h	297.48 MPa
k_h	336.72 MPa
l_h	585.50 MPa
m_h	380.53 MPa
n_h	707.13 MPa
o_h	307.45 MPa
p_h	588.56 MPa
q_h	557.68 MPa
r_h	425.23 MPa
s_h	430.63 MPa
t_h	598.90 MPa
h_average	438.76 MPa
h_deviation	125.36 MPa

Sample name	Shear Modulus
a_v	605.68 MPa
b_v	762.99 MPa
c_v	651.82 MPa
d_v	701.80 MPa
e_v	655.10 MPa
f_v	661.69 MPa
g_v	631.36 MPa
h_v	598.00 MPa
i_v	743.69 MPa
j_v	790.12 MPa
k_v	747.78 MPa
l_v	800.59 MPa
m_v	575.49 MPa
n_v	698.41 MPa
o_v	728.71 MPa
p_v	636.77 MPa
q_v	595.35 MPa
r_v	638.18 MPa
s_v	727.13 MPa
t_v	672.46 MPa
v_average	681.16 MPa
v_deviation	67.17 MPa

All_average	559.9586 MPa
All_deviation	157.8589 MPa



Load displacement relation for 3rd load cycle (less than 3N)



How to get the normal contact stiffness from the elastic shear modulus obtained

If it is assumed that the secant stiffness can be applied to the linear contact model, Eq.(1) with a certain level of the overlap δ_n gives the linear normal contact stiffness. The overlap level couldbe related to the maximum contact force level during simulation. The maximum contact force is calculated as 0.1N because of the material with 1111kg/m^3 of density and 0.1m of height.
So, the normal contact stiffness in the normal direction is calculated as,

$$(K_n)_{F=0.1N} = \left(\frac{2\langle G\rangle\sqrt{2\tilde{R}}}{3(1-\langle \nu\rangle)}\right)\sqrt{(\delta_n)_{F=0.1N}} \qquad (3)$$

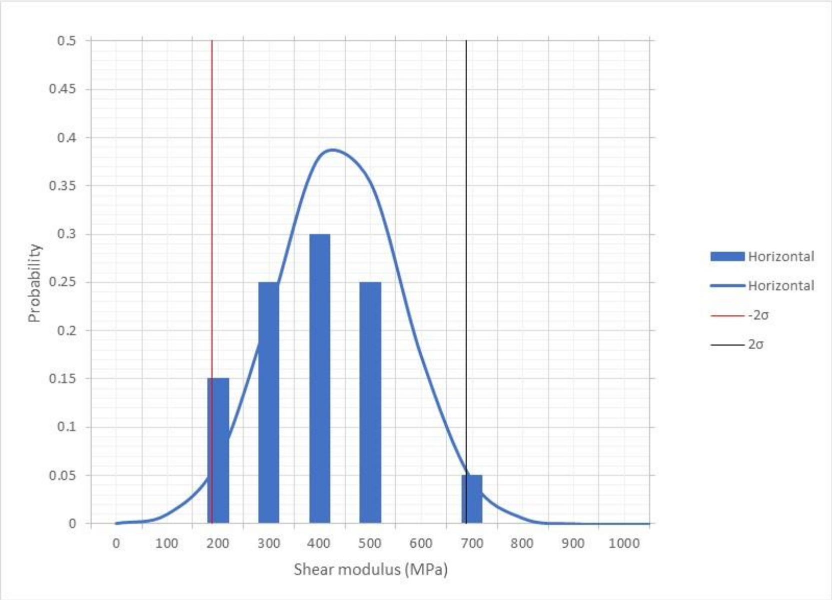
List of normal contact stiffness (for normal forces less than 0.1N)

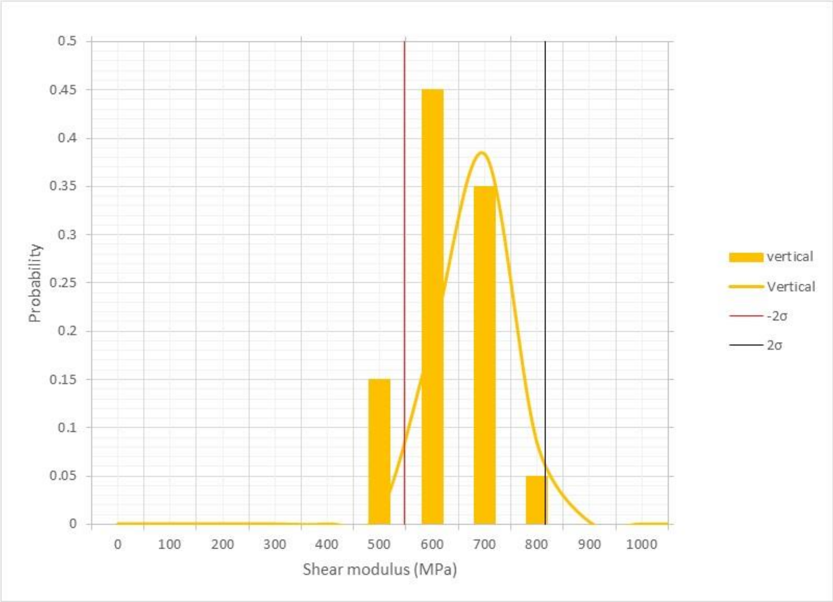
Sample name	Contact stiffness	
a_h	3.560 $\times 10^4$	N/m
b_h	4.882 $\times 10^4$	N/m
c_h	3.839 $\times 10^4$	N/m
d_h	4.537 $\times 10^4$	N/m
e_h	4.874 $\times 10^4$	N/m
f_h	4.803 $\times 10^4$	N/m
g_h	5.141 $\times 10^4$	N/m
h_h	6.235 $\times 10^4$	N/m
i_h	5.266 $\times 10^4$	N/m
j_h	5.061 $\times 10^4$	N/m
k_h	4.329 $\times 10^4$	N/m
l_h	6.270 $\times 10^4$	N/m
m_h	4.728 $\times 10^4$	N/m
n_h	7.136 $\times 10^4$	N/m
o_h	4.066 $\times 10^4$	N/m
p_h	6.272 $\times 10^4$	N/m
q_h	6.107 $\times 10^4$	N/m
r_h	5.058 $\times 10^4$	N/m
s_h	5.103 $\times 10^4$	N/m
t_h	6.365 $\times 10^4$	N/m
h_average	5.182 $\times 10^4$	N/m
h_deviation	0.945 $\times 10^4$	N/m

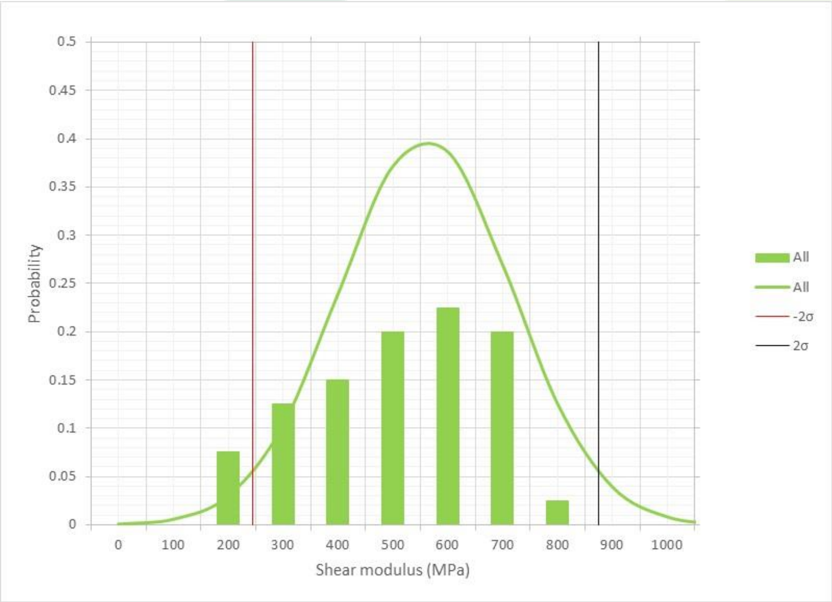
Sample name	Contact stiffness	
a_v	6.406 $\times 10^4$	N/m
b_v	7.472 $\times 10^4$	N/m
c_v	6.728 $\times 10^4$	N/m
d_v	7.067 $\times 10^4$	N/m
e_v	6.750 $\times 10^4$	N/m
f_v	6.795 $\times 10^4$	N/m
g_v	6.593 $\times 10^4$	N/m
h_v	6.355 $\times 10^4$	N/m
i_v	7.334 $\times 10^4$	N/m
j_v	7.648 $\times 10^4$	N/m
k_v	7.388 $\times 10^4$	N/m
l_v	7.706 $\times 10^4$	N/m
m_v	6.192 $\times 10^4$	N/m
n_v	7.030 $\times 10^4$	N/m
o_v	7.228 $\times 10^4$	N/m
p_v	6.606 $\times 10^4$	N/m
q_v	6.346 $\times 10^4$	N/m
r_v	6.599 $\times 10^4$	N/m
s_v	7.236 $\times 10^4$	N/m
t_v	6.865 $\times 10^4$	N/m
v_average	6.917 $\times 10^4$	N/m
v_deviation	0.453 $\times 10^4$	N/m

All_average	6.049 $\times 10^4$	N/m
All_deviation	1.143 $\times 10^4$	N/m

Distribution of elastic shear moduli values obtained







Distribution of normal contact stiffness values obtained