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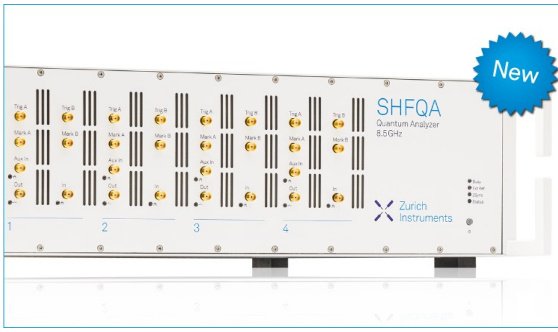
Takayuki Narumi, and Yoshiki Hidaka



Tagged-Particle Dynamics in Weak Turbulence: System Size Dependence of Nikolaevskii Turbulence

Takayuki Narumi[1,a)] and Yoshiki Hidaka[2]

[1]*Graduate School of Sciences and Technology for Innovation, Yamaguchi University, Ube 755-8611, Japan.*
[2]*Faculty of Engineering, Kyushu University, Fukuoka 819-0395, Japan.*

[a)]Corresponding author: tnarumi@yamaguchi-u.ac.jp

Abstract. Hierarchical interaction can cause turbulent flow even in a weakly nonlinear regime. One of such kinds of turbulence is weak turbulence, which is modeled as spatiotemporal chaos in nonlinear mathematical models. Here, we theoretically study tagged-particle dynamics in a model of weak turbulence, called Nikolaevskii turbulence; especially, the system size dependence is discussed in this paper. The system size does not affect the properties of anomalous diffusion when it is longer than a characteristic length. However, the properties with system size shorter than the characteristic length deviate from those with longer system sizes. It implies that there is a cutoff length in the system size of the Nikolaevskii turbulence. The cutoff length agrees with the critical system size already proposed in the discussion of dynamical exponents. Our results suggest that to investigate the Nikolaevskii turbulence at a low parameter region the system size must be set to be long enough.

INTRODUCTION

Turbulence is one of the challenging themes of physics. A lot of studies for turbulence have been devoted to fully-developed turbulence, which is caused by strong nonlinearities and shows the cascade mechanism. However, even in a weakly nonlinear regime, the interaction between several modes can cause disturbed flow due to the slaving principle. One of such kinds of turbulence is *weak turbulence* introduced by Manneville [1]. The weak turbulence is characterized as a turbulent flow in which laminar-like flow appears in a higher wavenumber region. For example, just above the threshold temperature difference of the convection-turbulence transition in thermal convective systems, pattern disorders globally while convective structure leaves in a small spatial scale. This regularity in small scale brings qualitative differences from fully-developed turbulence into weak turbulence. Here, we theoretically study Nikolaevskii turbulence, which is a theoretical model of weak turbulence in spatially extended nonlinear systems.

Let us consider evolutional equations represented by a function $G(\cdot)$

$$\frac{\partial \phi}{\partial t} = G\left(\frac{\partial \phi}{\partial x}, \frac{\partial^2 \phi}{\partial x^2}, \dots\right) \tag{1}$$

where $\phi = \phi(x, t)$ is a physical value characterizing pattern dynamics. We consider the long-wavelength phenomena so that higher-order terms are not significant. Because of the space reflection symmetry $x \to -x$, the evolutional equations must have even numbers of $\partial/\partial x$. The simplest approximation for Eq. (1) leads us to the Kuramoto–Sivashinsky (KS) equation [2]

$$\frac{\partial \phi(x,t)}{\partial t} + \left(\frac{\partial \phi(x,t)}{\partial x}\right)^2 = -\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^4}{\partial x^4}\right]\phi(x,t). \tag{2}$$

The solutions of the KS equation shows spatiotemporal chaos, called Kuramoto-Sivashinsky turbulence. The KS turbulence is well-studied as a theoretical model of weak turbulence in spatially extended systems. Recently, Bratanov et al. have proposed a multidimensional generalization of the KS equation [3]. Those models are fourth-order evolutionary equations. The dispersion relation of fourth-order equations has an extremum at most in the positive wavenumber region ($k > 0$). Thus, we need higher-order terms to study the interaction between distinguished modes. One of such

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equations is the Nikolaevskii equation [4]

$$\frac{\partial\phi(x,t)}{\partial t} + \left(\frac{\partial\phi(x,t)}{\partial x}\right)^2 = -\left[\frac{\partial^2}{\partial x^2}\left\{\varepsilon - \left(1 + \frac{\partial^2}{\partial x^2}\right)^2\right\}\right]\phi(x,t), \tag{3}$$

where ε is the control parameter. The Nikolaevskii equation has a neutral mode at $k = 0$ (i.e., Nambu–Goldstone mode); in addition, at $\varepsilon \geq 0$, the modes around $k = 1$ are unstable. Tribelsky and Tsuboi reported that the solutions of the Nikolaevskii equation with positive ε show spatiotemporal chaos [5]. The turbulent solution is called Nikolaevskii turbulence.

Although there are many works for the Nikolaevskii turbulence [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], the investigation from the Lagrangian description has not been done yet. We study the tagged-particle dynamics in the Nikolaevskii turbulence to understand it from the Lagrangian description [16]. In this paper, we present the system size dependence of the Nikolaevskii turbulence through the tagged-particle dynamics. The KS equation does not contain any control parameters; the system size dependence of the KS turbulence has been investigated [17]. Therefore, to investigate the system size dependence can lead to understanding the Nikolaevskii turbulence.

NUMERICAL METHODS

We numerically solved Eq. (3) with the system length L and the space increment $\Delta x = 2^{-1}$. The boundary condition was the periodic boundary condition for x-direction. The equation was numerically solved by a pseudo-spectral method. The time-stepping was done by the exponential time differencing scheme of the second-order Runge–Kutta methods [18] with the time increment $\Delta t = 10^{-2}$. The initial condition of ϕ was the perturbation around $\phi = 0$, and the waiting time before sampling was $t_w = 10^5$.

We regard the short-wavelength modes around $k = 1$ as modes relating to convection and then introduce the z-axis with $0 \leq z \leq \pi$. Sakaguchi assumed that convective velocity field (u, w) from $\phi = \phi(x, t)$ [19];

$$(u(x,z,t), w(x,z,t)) = \left(-\frac{\partial\phi}{\partial x}\frac{dF}{dz},\ \frac{\partial^2\phi}{\partial x^2}F\right), \tag{4}$$

where $F(z) = (2z/\pi)^2(2 - 2z/\pi)^2$ is a function characterizing convection with the rigid boundary condition of the z-direction. Figure 1 shows the snapshots of the solutions $\phi(x, t)$ and the corresponding velocity fields (u, w). It is confirmed that the convection structure on the x-z plane successfully obtained from the one-dimensional solution $\phi(x, t)$ of the Nikolaevskii equation.

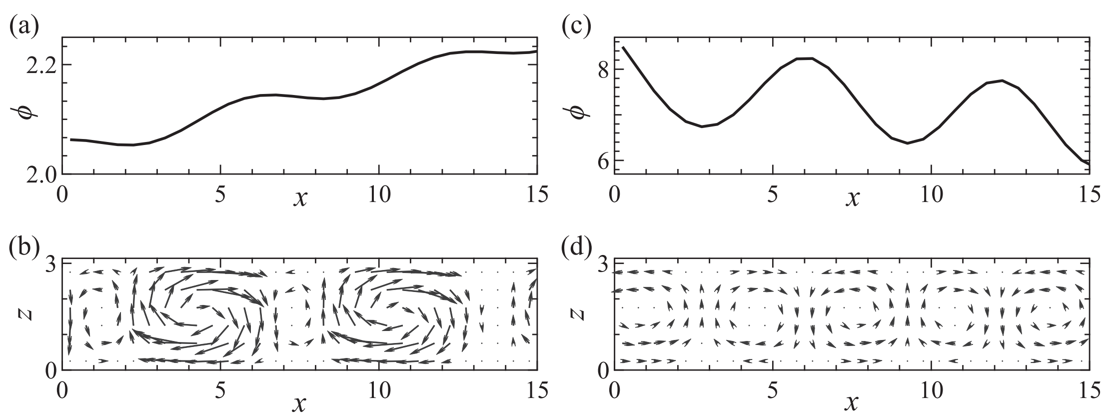

FIGURE 1. Velocity field of the Nikolaevskii turbulence. The values of the control parameter ε are 0.01 at (a) and (b), and 0.1 at (c) and (d). The upper figures show snapshots of the solutions $\phi(x, t)$, and the bottom ones do the corresponding velocity fields (u, w). The arrow length in (b) and (d) indicates the relative speed of the fluid velocity; the speed increases with increasing ε.

Under the conditions that the small mass of the tagged-particle, the tagged-particle velocity $(V_x(t), V_z(t))$ is equivalent to the velocity field, that is,

$$(V_x(t), V_z(t)) = (u(X(t), Z(t), t), \quad w(X(t), Z(t), t)), \tag{5}$$

where $(X(t), Z(t))$ denotes the position of the tagged-particle.

Diffusive property can be characterized by the mean-square displacement;

$$M_2(\tau) = \left\langle |X(t+\tau) - X(t)|^2 \right\rangle, \tag{6}$$

where we only considered the displacement on the x-direction. Diffusion can become anomalous in complex systems. Such anomalous diffusion is characterized by the anomalous parameter γ, defined in $M_2(\tau) \propto \tau^{\gamma+1}$. The timescale dependence of γ is calculated as

$$\gamma(\tau) = \frac{d \log M_2(\tau)}{d \log \tau} - 1. \tag{7}$$

Note that the anomalous parameter for the conventional Brownian motion decreases from the ballistic motion (i.e., $\lim_{\tau\to 0} \gamma(\tau) = 1$) to the normal diffusion (i.e., $\lim_{\tau\to\infty} \gamma(\tau) = 0$) monotonically.

RESULTS AND DISCUSSION

Figure 2 presents the size dependence of the anomalous parameter $\gamma(\tau)$ for several parameters in $0.0001 \leq \varepsilon \leq 0.04$. First, at $\varepsilon = 0.04$ (Fig. 2 (a)), the results for $L \geq 512$ almost overlap. Although the times at which $\gamma(\tau)$ has an extremum are identical even in the result of $L = 256$, the extreme values are slightly lower than the others. It implies that there is a characteristic system size L_c of the Nikolaevskii turbulence which is $256 < L_c < 512$ at $\varepsilon = 0.04$. Next, at $\varepsilon = 0.02$ (Fig. 2 (b)), in addition to the result of $L = 256$, the result of $L = 512$ also departs from the longer-size results. The results of $L < L_c$ disagree with each other, while those for $L > L_c$ overlap. It suggests that the characteristic size is a kind of a cutoff length. We thus should study the system size larger than L_c to understand the Nikolaevskii turbulence. Tanaka and Okamura mentioned that a critical system size is within $256 < L < 512$, according to their results of the dynamic exponent [15]. Since it is hard to think that there are multiple critical points in a narrow length range, we consider their critical size and our characteristic size as being identical. The characteristic size L_c increases with decreasing ε. For instance, when one studies the Nikolaevskii turbulence for $\varepsilon \geq 0.001$, the system size must be larger than the characteristic size larger than $L \simeq 4096$ as shown in Fig. 2 (f).

SUMMARY

To study the Nikolaevskii turbulence from the Lagrangian description, we numerically solved the one-dimensional Nikolaevskii equation (3) and carried out a tagged-particle simulation dominated by the turbulent flow, which is obtained from the solution of the Nikolaevskii equation. Especially, we focus on the system size dependence of the anomalous diffusion properties characterized by the anomalous parameter γ. The system size does not affect the properties of anomalous diffusion when it is longer than a characteristic length L_c. However, the results of system size shorter than L_c deviate from those of sizes longer than L_c. It implies that L_c can be regarded as a cutoff length in the system size of the Nikolaevskii turbulence. The cutoff length agrees with the critical system size proposed by Tanaka and Okamura [15]. To investigate the Nikolaevskii turbulence at a low ε region, our results suggest that the system size should be lengthened than L_c. In this paper, the origin of the characteristic length L_c has not been discussed. We now try to understand it from a simple theoretical model [16].

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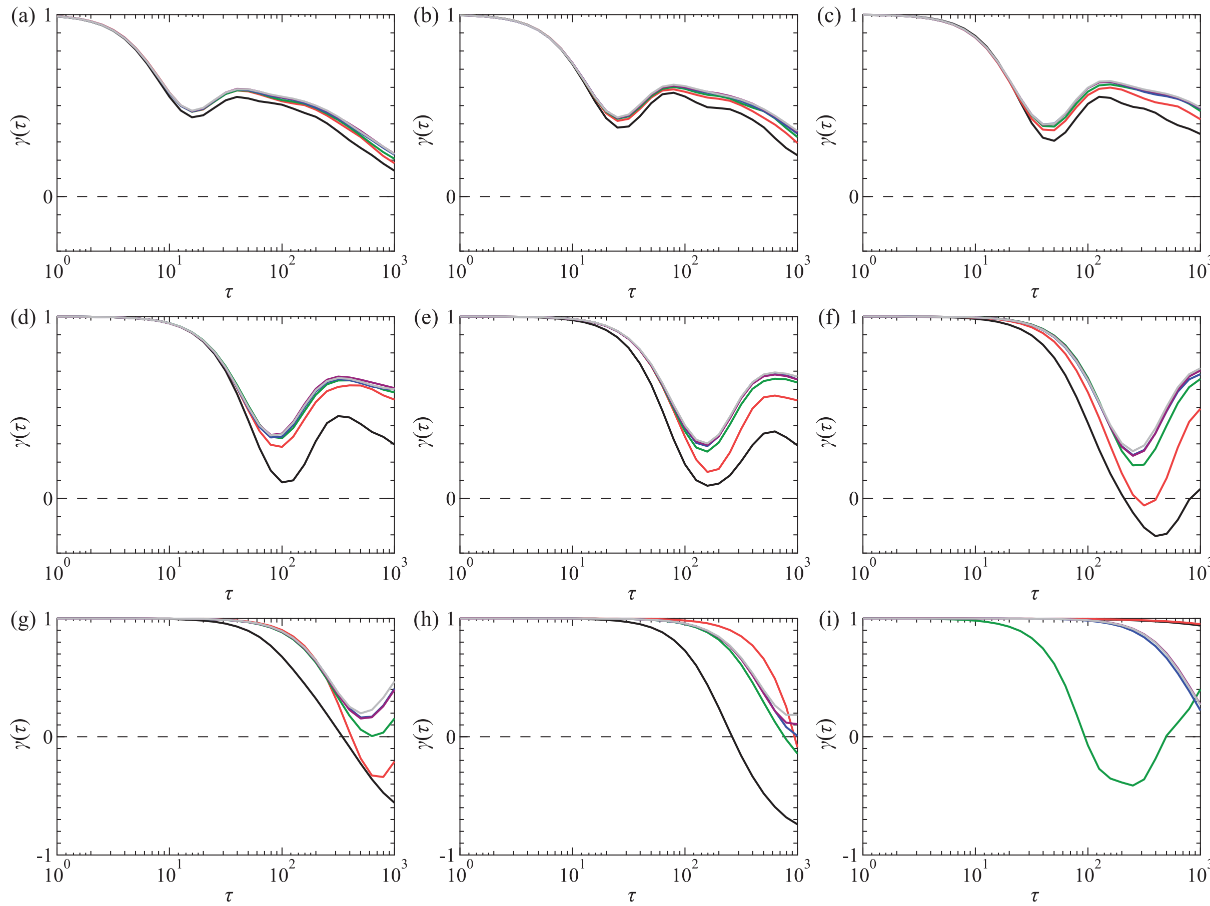

FIGURE 2. Size dependence of the anomalous parameter $\gamma(\tau)$ at $\varepsilon = 0.04$ (a), 0.02 (b), 0.01 (c), 0.004 (d), 0.002 (e), 0.001 (f), 0.0004 (g), 0.0002 (h), and 0.0001 (i). Each panel includes results for several system sizes: $L = 256$ (black), 512 (red), 1024 (green), 2048 (blue), 4096 (purple), and 8192 (gray), from bottom to top.

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