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AN AGENT-BASED MODEL FOR UNDERSTANDING SYMMETRIC ALIGNMENT OF HONEYCOMB

TAKAYUKI NARUMI, KENTA UEMICHI, HISAO HONDA AND KOICHI OSAKI

Name: Takayuki Narumi (b. Osaka, Japan)
Profession: Lecturer (Yamaguchi University)
Fields of interest: Nonlinear Nonequilibrium Physics, Hierarchy structure in nature
Address: Tokiwadai 2-16-1, Ube, Yamaguchi, 755-8611, JAPAN.
E-mail: tnarumi@yamaguchi-u.ac.jp
Home-page: http://web.cc.yamaguchi-u.ac.jp/~tnarumi/index.html
Awards: Prize for Encouragement of Society for Science on Form
Publications and/or Exhibitions:
Narumi,T., Uemichi, K., Honda, H., and Osaki, K. (2018) Self-organization at the first stage of honeycomb construction: Analysis of an attachment-excavation model, *PLoS ONE*, 13, No.10, e0205353, doi.org/10.1371/journal. pone.0205353.

Name: Kenta Uemichi (b. Wakayama, Japan)
Profession: Ph.D student (Kwansei Gakuin University)
Fields of interest: Mathematics, Mathematics Education
Address: Gakuen 2-1, Sanda, Hyogo, 669-1337, JAPAN.

Name: Hisao Honda (b. Kyoto, Japan)
Profession: Professor (Kobe University[1]), Researcher (Center for Biosystems Dynamics Research, RIKEN[2])
Fields of interest: Morphogenesis, Theoretical Biology
Address1: Kusunoki-chou 7-5-1, Chuou-ku, Kobe, 650-0017, JAPAN.
Address2: Minami-machi 2-2-3, Minatojima, Chuou-ku, Kobe, 650-0047, JAPAN.

Name: Koichi Osaki (b. Hiroshima, Japan)
Profession: Professor (Kwansei Gakuin University)
Fields of interest: Nonlinear Analysis for Reaction-Diffusion Systems
Address: Gakuen 2-1, Sanda, Hyogo, 669-1337, JAPAN.
Home-page: https://sci-tech.ksc.kwansei.ac.jp/~osaki/

Abstract: *Honeybees construct their nests that consist of symmetrically arrayed hexagonal cylinders. In the first stage of honeycomb construction, they build a linear sequence of tetrapod structures that form the basis of honeycomb. However, it is unknown how honeybees produce that initial pattern. Herein, to understand the mechanisms of honeycomb construction, we propose an agent-based model, the attachment-excavation model, in which worker honeybees are classified into attachers and excavators. We then conduct two-dimensional simulations that show how a tripod pattern can be seen as a projection of tetrapods onto a plane. The simulation results show that the tripod pattern* *emerges due to competition between the attachers and excavators. As time advances, the isotropic wax growth causes the tripods to connect planarly. Further, we employ anisotropic wax growth to obtain a linear sequence of constructed tripods, thus suggesting that anisotropy is a significant contributor to the first stage of honeycomb construction. From our simulation results, we conclude that honeybees utilize self-organization to achieve complexity during the first stage of honeycomb construction.*

1 SYMMETRIC ALLIGNGMENT ON HONEYCOMB

Complex patterns can appear without the use of top-down methods. One bottom-up approach is the self-organization formation process of higher-level order that spontaneously arises out of local interactions among lower-level components. Living organisms show numerous self-organized pattern types, such as the growth of bacterial colonies, the synchronized light emissions of fireflies, and the swarm dynamics of fish and birds (Yates, 1987; Camazine, 2001; Dobrescu, 2011). Some construction processes involving social insects such as ants (Franks, 1997) and termites (Deneubourg, 1977) can be at least partially explained in terms of self-organization.

Western honeybees are a leading example of social insects. They live communally and care cooperatively for their young. The structure of honeybee nests consists of double-sided regularly spaced cavities. The axes of these cavities appear to be almost horizontal (Tautz, 2008). Each hole is created in the form of a precise hexagonal prism (Fig. 1). Honeybee nest construction, including the process by which honeycombs are made from the wax secreted by worker honeybees, has long attracted scientific interest (Darwin, 1859).

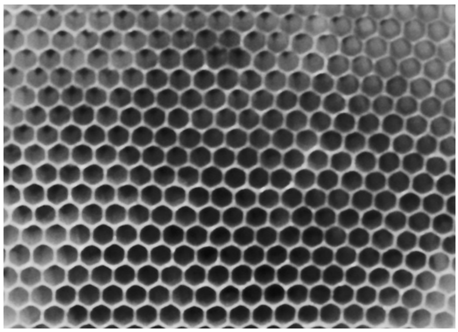

Fig. 1: Hexagonal patterns on honeycomb.

Honeybees have long been considered capable of self-organization. For example, a swarm of honeybees controls its temperature as needed to reflect changes in the environment (Heinrich, 1981a, 1981b), and such thermoregulation in the honeybee swarm has been modeled by self-organization (Myerscough, 1993; Watmough, 1995). Honeybee colonies search for nectar sources within their foraging range and choose better ones (Seeley, 1995), and a modeling approach has shown that the efficient concentration of effort can be interpreted as a self-organized mechanism (Camazine1991a). Furthermore, comb pattern usage itself can be described by a bottom-up process (Camazine, 1990, 1991b; Jenkins, 1992; Johnson, 2009; Montovan, 2013). As in the honeycomb construction process, it has been noted that honeybees benefit from self-organization (Tautz, 2008; Belić, 1986; Škarka, 1990). We consider that honeybees are engineers utilizing self-organization even in the first process of honeycomb construction. We should then configure simple behavioral rules of honeybees to study honeycomb pattern formation.

The mechanism by which honeybees construct honeycomb cells in such precise order is still an open discussion. To understand the origin of these regular arrangements, we focus our attention on the initial structure of the honeycomb. In the early phase of the construction process, the workers on the ceiling make tetrapod structures (Fig. 2). These structures can be treated as the basic building block in honeycomb construction. Therefore, a clarification of the tetrapod formation mechanism can be expected to shed light on the first stage of the honeycomb construction process.

Fig. 2: A tetrapod structure on ceiling, as the basic building block in honeycomb construction.

2 ATTACHEMENT-EXCAVATION MODEL

To clarify the tetrapod construction from a self-organization viewpoint, we have proposed an agent-based model --- the *attachment-excavation model*--- in which the roles of worker honeybees are modeled into the growth of beeswax and the dynamics of excavators (Narumi, 2018). Since the workers act according to simple rules, our model does not require them to have any prior knowledge of the complex shape that they build. We carried out the numerical simulation for our agent-based model and obtained the tripod structure, which is the basic building block of the honeycomb structure in a two-dimensional reduction, resulted by the competition between the wax-attaching and wax-removing workers. Thus, the tripod structure can be regarded as a dissipative structure. In addition, by supplying wax unidirectionally, the anisotropic connection of tripod patterns has been also obtained.

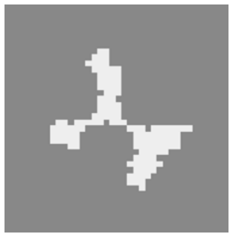

Fig. 3: A tripod structure in the attachment-excavation model.

We can conclude that the first stage of honeycomb construction can be understood in terms of self-organization, the formation of tetrapod structures (dissipative structure), and their one-dimensional connections (self-assembly). After the first stage, it is speculated that the basic building blocks are set up in honeycomb construction as shown in Fig.4. The three-dimensional simulation of the attachment-excavation model will be effective for understanding how the symmetric hexagonal pattern emerges from a viewpoint of the attachment-excavation model.

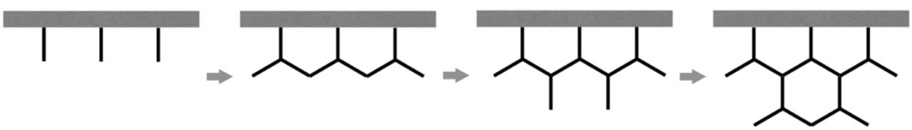

Fig. 4: A sketch of development of honeycomb to the vertical direction.

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